

# Libertarian free will and quantum indeterminism

Chetan S. Mandayam Nayakar\*, S. Omkar, R. Srikanth†

Poornaprajna Institute of Scientific Research, Sadashivnagar, Bangalore, India and  
Raman Research Institute, Sadashivnagar, Bangalore, India

The basic question in the long-standing debate about free will (FW) is not whether FW can be demonstrated to exist nor even whether it exists, but instead how to define it scientifically. If FW is not dismissed as an illusion nor identified with a variety of unpredictability, then logical paradoxes arise that make FW elusive to define. We resolve these paradoxes through a model of FW, in which FW is a new causal primitive empowered to override physical causality under guidance. We develop a simple mathematical realization of this model, that when applied to quantum theory, suggests that the exercise of FW corresponds to a nonlinear POVM causing deviations from the Born rule. In principle, these deviations would stand in conflict with known conservation laws and invariance principles, implying that the brain, the presumed seat of FW, may be an arena of non-standard physics. However, in practice it will be difficult to distinguish these deviations from quantum and neural noise, and statistical fluctuations. We indicate possible neurobiological and neurological tests, implications and applications of our proposed model.

## I. INTRODUCTION

Free will (FW), as we informally understand it in daily life, is the power of a rational agent to pick her/his own choice from among various alternative possibilities. But what exactly is FW? The age-old question has provoked much debate among philosophers and scientists [1–4]. Our outlook on the world implicitly assumes that human behavior is governed by FW: we choose, we plan and we normally hold people responsible for what they say or do, either because we imagine that they are at liberty also not to act so, or because we don’t have the freedom to believe otherwise! Neuroscience has been uncovering causal chains that appear to explain our choices, emotions and even beliefs, in terms of neurophysical events extending back to the pre-natal stage. As much as it is interesting and important to understand whether we have FW, there is a more elementary and pressing problem to deal with: namely to try to define FW rigorously and scientifically in a way that agrees with the above intuitive notion. This last requirement is important, as one can define almost anything one wants in a theory of one’s own design [5]. Having defined FW, one might ask whether such a FW is compatible with known scientific knowledge or can be demonstrated experimentally as a new kind of resource in Nature.

Philosophers over the millenia have proposed different responses to the problem. Two prominent positions are *compatibilism*, according to which determinism is compatible with FW, and *incompatibilism*, according to which the two are incompatible. There are shades of intermediate positions. Compatibilism is espoused by, among others, Calvinists who believe that personal freedom to choose does not preclude foreknowledge (possessed by a all-powerful intelligence) of future choices. Two broad incompatibilist positions are (*metaphysical*) *libertarianism* and *hard determinism*. According to the former, in a situation of alternative possibilities (AP) which is ontologically (metaphysically) available, an agent has the liberty to choose one or other option, which is not pre-determined. This position upholds FW, and rejects determinism. By contrast, *hard determinism* upholds determinism and rejects FW, relegating it to an illusory feeling. In Western philosophical thought, this is represented by, among others, Armenianism.

Recently, a number of physicists have studied FW mainly (but not exclusively) in connection with quantum mechanics (QM) [6–19]. The important contribution of quantum mechanics to this debate is in introducing a concrete instance of fundamental indeterminism via the  $|\psi|^2$  Born rule. Whether this indeterminism is epistemic or ontic still remains a moot issue and is related to the notion of realism in the sense of the Bell-Kochen-Specker theorems. In the quantum approach to FW, some have assumed that quantum indeterminacy gives ‘elbow room’ for FW to act, while others identify FW with unpredictability, or independence from all past information (with ‘past’ defined in an appropriately relativistically invariant way). We call this approach *soft incompatibilism*, according to which FW is incompatible with determinism but compatible with indeterminism. However, this approach, which also finds favor in the neuroscientific and artificial intelligence (AI) communities, fails to capture the sense of control, of having liberty and making a deliberate choice, implicit in the notion of FW.

---

\* aka M. N. Chetan Srinivas

†Electronic address: srik@poornaprajna.org

In this work, we aim to define libertarian FW, which encompasses such a liberty, and to study its relation to the laws of physics. Libertarian FW is arguably the most intuitive version of FW, and yet, as we will find, the most elusive to define. We do not claim that it is somehow a better variety of FW, or that it can be experimentally demonstrated at this time. It is our preferred understanding of FW simply because it feels right! The article is arranged as follows. Section II discusses a specific compatibilist version of FW, due to the cognitive scientist Mackay. Section III presents a logical paradox that would afflict attempts to define libertarian FW, and presents a resolution. Section IV explains another basic logical paradox about libertarian FW. This logical barrier is surmounted in Section V through a new model of FW. Certain biological and philosophical aspects of the model are considered in Section VI. Sections VII and VIII may be omitted by readers not interested in quantitative aspects of the model. Section VII presents a simple mathematical representation of the proposed model, while Section VIII applies it to quantum mechanics and considers some unusual consequences. Neurological tests, implications and medical applications of our model are considered in Section IX. We finally conclude with Section X, with a discussion on FW in connection with Darwinian evolution and the subject of consciousness.

## II. THE MACKAYAN ARGUMENT: FREE WILL FROM UNCOMPUTABILITY

We mention two flavors of what are arguably compatibilist positions: Calvinist-like (mentioned above) and Mackayan. In the Mackayan position [20], the idea here is that even in deterministic dynamics, self-predictions lead to indeterminacy, which is interpretable as FW.

According to this argument, given prediction  $P$  about agent  $X$ , if  $X$  believes  $P$ , then she can falsify it by acting contrary to  $P$ . One might suppose that a more detailed algorithm to arrive at  $P$  would be able to take into consideration  $X$ 's reaction, but  $X$  can simply choose to falsify *that*. It then follows that only if  $X$  disbelieves  $P$ , then  $P$  holds with certainty, but then this would make  $X$  a kind of inconsistent agent, for disbelieving a truth. Thus, if  $X$  (and hence the universe) is consistent, then  $X$  must believe  $P$ . To avoid the first inconsistency, that she falsify  $P$  if she knew it, we conclude that believing  $P$  affects  $X$ 's brain state, invalidating the premise that went into reaching  $P$ . Thus,  $X$ 's falsification of  $P$  does not imply inconsistency. But it does imply that any consistent predictive algorithm  $P$  is unable to encompass her reaction to  $P$ . Therefore, according to Mackay, it is upto  $X$  how  $X$  chooses to act; she can make  $P$  or  $\neg P$  true. This indeterminability is interpreted as FW.

A careful scrutiny reveals the similarity of the structure of the above argument analogous to Gödel's celebrated incompleteness theorem [21], when the notion of *provability* is replaced with that of *belief*, though, strangely, Mackay denied the connection. To recast the Mackayan position in a form closer to Gödel's theorem: in a consistent sufficiently rich, deterministic world, there will be unpredictable situations. Not surprisingly on account of the presence of self-reference, Mackay's argument can also be compared readily to Turing's proof of the unsolvability of the halting problem, where the putative algorithm that outputs  $P$  is analogous to the would-be halting algorithm  $h$  and the nay-saying agent to the contradictory Turing machine that uses the  $h$  as a sub-routine. Mackayan free will, like Turing uncomputability, comes from the fact that the cardinality of the set of outcome situations is greater than proofs of prediction of an agent's choice. Thus, in the Mackayan argument, unpredictability of a free agent's choice is identified with Gödel incompleteness and Turing uncomputability.

But does incompleteness or uncomputability constitute a kind of FW? Intriguing though the idea is, we think that it is, ultimately, philosophically unsatisfactory, because it identifies free will with unpredictability rather than with genuine freedom, or even indeterminacy. Likewise, from a philosophical perspective, we think that the soft incompatibilist approach from quantum mechanics does not go far enough, because sense of control in FW is not explained. FW in its various quantum avatars looks just like free whim!

## III. THE WEAK FREE WILL PARADOX AND ITS RESOLUTION

The reason libertarian FW resists definition is that attempting to define it leads to logical paradoxes, which we discuss in this and the next section. Although these paradoxes were not, to our knowledge, explicitly mentioned before, yet they implicitly crop up in past attempts to define FW, thwarting a definitive resolution to the problem for over two millenia. It is not surprising that in the face of this daunting logical barrier, scientists tend to adopt a position other than libertarianism.

If we accept the libertarian position, then by assumption of incompatibilism, one rules out determinism as the universally correct description of the physical laws. But neither does indeterminism (as governed by some fixed probability rule,  $P$ ) leave enough room for FW to act. Consider the sample mean  $X_n$  over  $N$  trials

$$\lim_{n \rightarrow \infty} \Pr(|\bar{X}_n - \mu| > \epsilon) = 0,$$

by the Weak Law of Large Numbers. This implies that there is a ‘probability pressure’ *not* to choose *atypical* sequences, which would cause deviations from the sample mean. Thus, there is a kind of long-run determinism, and hence a restriction on FW as we intuitively understand it. For example, given a coin with outcome space  $\Omega = \{H, T\}$  and the corresponding probability vector  $P = (\frac{1}{2}, \frac{1}{2})$ , the coin is not free to indefinitely choose outputs HHHHHHHHH... Thus libertarian FW is compatible with neither determinism nor indeterminism, and thus belongs to the position of *hard incompatibilism*. This situation creates a paradox for the libertarian position, which we call the Weak FW Paradox (WFWP), whereby FW is compataible with neither of the available alternatives. The other mentioned positions on FW are unaffected by WFWP.

Therefore, if we accept libertarian FW, the free-willed choice will potentially interfere with the underlying physical dynamics  $\mathcal{D}$ . This interference will take the form of (a) overriding causality, if  $\mathcal{D}$  is deterministic; or (b) causing deviations from the relevant probability rule  $P$ , if  $\mathcal{D}$  is indeterministic.

Clearly, it is immaterial whether the underlying physics is classical or quantum. The argument sometimes made, that quantum indeterminism gives ‘room’ for free-willed action is thus not found to be persuasive. FW itself can’t be part of the dynamics  $\mathcal{D}$ , for in that case it could not produce the required deviations. Therefore, we require a Cartesian dualism with a physical and *extra-physical* component making up a free-willed agent. Physical dynamics  $\mathcal{D}$  governs the former while FW comes from the latter.

#### IV. THE STRONG FREE WILL PARADOX

The above resolution of WFWP says that if libertarian FW exists, it must be an extra-physical resource whose intervention causes deviations from  $\mathcal{D}$ . It still leaves open the question what libertarian FW is or how it can be consistently defined. Here we will show how trying to pin down its nature leads to another paradox, which we call the Strong Free Will Paradox (SWFP). The paradox is obtained essentially by extending the traditional incompatibilist claim for the incompatibility of determinism and FW, to that of *indeterminism* and FW. The idea is that the truth of determinism or indeterminism would mean that we don’t control actions in a way one would expect of self-determining, free-willed agents. Since determinism and indeterminism are the only available causal primitives, there is no such thing as libertarian FW, according to this line of reasoning. There is a similarity to WFWP, but the focus has shifted from “where?” to “what?”, or even “whether?”.

We present two slightly different versions of SWFP. In the first of them, one assumes that FW exists and tries to locate it in terms of properties of the agent. According to this argument:

- [I] An agent has free will only when her intentional actions emerge from the agent herself.
- [II] Therefore, they are deterministic functions of her volitions, beliefs, desires etc, which we denote by *variables of intent*,  $x_j^{(1)}$ .
- [III] If there are no prior causes of her volitions, desires, etc., then the values  $x_j^{(1)}$  take on must be random, making the agent’s volitions, desires, etc., and hence her actions, whimsical.
- [IV] Since whimsicality undermines the notion of control or intent,  $x_j^{(1)}$  must not be random, but deterministic functions of some other second order intent variables  $x_k^{(2)}$ , which, by virtue of [I], must belong to the agent herself.
- [V] As before, the variables  $x_k^{(2)}$  themselves are caused or uncaused. If the latter, then  $x_k^{(2)}$  are random, making  $x_j^{(1)}$ , and hence also her actions, ultimately whimsical. But if the former, then  $x_k^{(2)}$  are not free but depend, by a similar reasoning, on higher order variables  $x_j^{(3)}$ , and so on similarly to even higher orders indefinitely.
- [VI] If this pattern of recursion terminates at some finite depth  $N$ , then  $x_j^{(N)}$  either has no causes or external causes of unspecified origin. In the former case, we obtain capricious, indeterministic behavior, in the latter case, unfree, deterministic behavior. Neither connotes FW as we recognize it.

In brief, when we try to incorporate the notion of will or intent into the action, the action becomes deterministic and hence unfree. Putting freedom back means removing determinism, which undermines intent by making the action random and the agent whimsical. Thus the ‘free’ and the ‘will’ in ‘free will’ are at loggerheads, making the word an oxymoron. This is the Strong Free Will paradox (SWFP), according to which randomness and determinism seem to be the only fundamental causal primitives in Nature, with libertarian FW a figment of imagination.

It is of interest to note that certain classical accounts of FW fall prey to SWFP, illustrating its elusiveness to define. According to Thomas Hobbes, “A free agent is he that can do as he will, and forbear as he will....”. David Hume

characterizes it thus: “power of acting or of not acting, according to the determination of the will: that is, if we choose to remain at rest, we may; if we choose to move, we also may.... This hypothetical liberty is universally allowed to belong to everyone who is not a prisoner and in chains.” To paraphrase in terms of our discussion above, they both are essentially saying  $E = E(x_j^{(1)})$ , but weren’t taking the argument farther. Arthur Schopenhauer does take it one step farther but says: “You can do what you will, but you cannot will what you will. In any given moment of your life you can will only one definite thing and absolutely nothing other than that one thing.” He is thus effectively responding to SWFP by characterizing  $E = E\left(x_j^{(1)}(x_k^{(2)})\right)$  and denying the existence of libertarian FW. One can try other variants, such as  $E = E(x_j^{(1)})$  being a probabilistic function, whereas  $x_j^{(1)} = x_j^{(1)}(x_j^{(2)})$  being deterministic, so that there is mix of determinism (Will) and indeterminism (freedom) at *different* levels of the agent’s personality. However, the core of SFWP remains.

A related, but different version of SFWP, tries to identify FW as a particular kind of influence over the act of making a choice. According to this argument:

- [A] Suppose that from a set of alternative possibilities (AP), a choice is eventually made by an agent.
- [B] From the principle of excluded middle, the choice was made either according to a rule or it was not made according to a rule.
- [C] In the former case, there is no genuine AP situation. Hence we have determinism, and there is no FW.
- [D] In the latter case, we have pure randomness, and hence, again, no FW in the sense of the agent’s control and voluntary choice.

We thus find again that the agent’s choice is deterministic or random, with no apparent room for (libertarian) FW. This is the basis of the *pessimist* view that there can be no such thing as FW [2].

Implication [C] lies historically at the heart of the incompatibilist argument. Here, if the rule is *prescriptive*, we have *causal determinism*, as a law of physical dynamics in Nature. If the rule is only *descriptive*, we have *logical determinism*, with outcomes of future choices being known to an all-powerful intelligence and assigned definite truth values.

## V. RESOLUTION OF THE STRONG FW PARADOX

We propose the following hard incompatibilist model as a way out of SFWP, drawn from an interpretation of Eastern philosophy. According to the model, the conscious personality of the free-willed agent, which may be called the *Ego* (after the father of modern psychoanalysis, Sigmund Freud), is influenced in an AP situation by three structural elements. This tripartite division is at the heart of our model. Two of the elements, and their functions, are as follows:

**Nature (N).** Imposes mental constraints in the form of desires, instinctive drives and emotional tendencies.

**Understanding (U).** Offers guidance in action through a rational capacity to model the world, and understand the (ethical, social, financial, etc.) implications of each choice.

When a situation with alternative possibilities is encountered, Nature **N** and Understanding **U** present their respective recommendations to the Ego on selecting one of the alternatives. While the recommendation of Nature appears as a *desire*, that of Understanding appears as a *thought* or *feeling*. As a specific example, the desire may be oriented towards seeking pleasure (Freud’s pleasure principle), while the thought or feeling may be about conscientious restraint. The dilemma sometimes experienced by one about to make a choice is the possible conflict between the two recommendations.

It is convenient to designate the *mind* as the seat of Nature **N**. In Freudian psychoanalytic terminology, the mind may be identified with the *id*. Similarly, the seat of **U** is posited to be the *intellect*, which may be identified with Freudian *superego*. To use a crude but visually helpful computer analogy, we may picture the ego as the CPU (central processing unit) of a computer, with **U** and **N** being softwares uploaded onto it during the waking period from their respective seats, which are like hard disk memory locations.

For non-libertarian accounts of FW, including AI, the two elements **U** and **N** suffice. They are both *causal resources*, i.e., systems that can drive the agent’s behavior independently, and thus aspects of the dynamics  $\mathcal{D}$ . The eventual choice will be some deterministic or probabilistic outcome of their interplay. Philosophically speaking, the main drawback with such accounts is that they lack the sense of having liberty to make a deliberate choice.

Crucially, for (libertarian) FW, **U** is not a causal resource, but a *guidance resource*, unlike **N**. When the input from **U** and **N** are experienced as possibly conflicting tendencies by the Ego, the guidance by itself is powerless to influence the agent’s choice. A further element is required, the third and last in our model, which expresses the idea of empowering the Ego to deliberately choose between the recommendations of desire and guidance. This is the faculty of volition, or simply:

**Freedom of Will (F).** The *extra-physical* freedom to orient or align the choice in line with the Understanding **U**, by overcoming, if necessary, the Constraints imposed by Nature **N**.

The extra-physicality follows from the resolution of the WFWP. SWFP also implies it because if it were not extra-physical, then **F** would be part of the underlying dynamics **D** as a deterministic or indeterministic aspect, and not libertarian, as required. As to the question of whether such an extra-physical resource is sanctioned by physics or can be tested, we return to it in Section VIII. Here we are concerned only with obtaining a suitable *definition*. We call our model ‘FUN’ in recognition of the fact that it involves the triad of elements **F**, **U** and **N**.

Faced with an AP situation, Nature and Understanding may present conflicting recommendations to the Ego under some description— a desire or drive vs a thought, feeling or belief. If the Nature-induced desire is aligned away from the Understanding-proposed thought, then supported by FW, the thought acts as a restraint, whereas if the desire is attuned to **U**’s recommendation, then FW acts to that extent effortlessly. Together **F** and **U** constitute a causal resource, in which **F** contributes the magnitude or “energy” while **U** the direction. Recognizing this is crucial in resolving SFWP, and its oversight is the cause of much confusion in existing accounts of FW. Thus FW is the expression of **F** in a situation where the ego is exposed to the potentially conflicting recommendations of **U** and **N**.

FW as defined above is the new *causal primitive* or *principle of causation*, apart from determinism and indeterminism. By its fundamental character, it is empowered to override physical causality (of Nature) under the intellect’s guidance. FW thus enables a state of affairs wherein logical determinism holds whereas causal determinism fails. We return to this point elsewhere to use this as a basis for a (meta-)logical framework to characterize FW. We think that our definition of FW, and a quantification of it below, agree with the notion of *volitional causation* proposed in Ref. [25], as distinguished from *physical causation*.

The agent’s freedom to control, together with the Understanding, as we have proposed, bears some similarity to the *Ultimate Responsibility* posited by the philosopher Kane [26]. However, the role of **F** and **U** need not be confined to moral issues alone, and may extend to other walks of life. For example: agent *X* is suffering from a medical problem, and has the Understanding that *X* needs to undergo treatment. However *X*’s Natural tendency, governed by Constraints **N**, would be to avoid the treatment because it would cause discomfort. *X* exercises FW to endure the discomfort in the interest of later cure and longterm benefits.

We will now prove that the FUN model resolves SFWP. In particular, the above example highlights how the *incompatibilist* implication [C] of SFWP fails. If one happens to be familiar with the *Weltanschauung*, and hence the Understanding of *X*, and *X* has high free will, then the *freely* chosen alternative will be *predictable*. Proposition 1 below, whose truth is clear in this light, is rigorously validated by our definition of FW. We define a *Saint* as an agent whose Understanding is ethical and whose FW is near maximal.

**Proposition 1** *Presented with the choice between the good and the evil, the Saint freely chooses the good.*

By definition, the Saint, equipped with maximal FW, can if necessary override any Constraints like desire and instinctive drives imposed by his human nature, and align his choice in tune with his Understanding.

Thus, the deterministic behavior and predictability of the Saint does not preclude his free will, contrary to the *incompatibilist* implication [C] in SWFP. Knowing that a person is a Saint, we have sure *foreknowledge* of the ethicality of all his future choices, *because* he is endowed with high FW. We have here an instance of a descriptive rule (logical determinism) that is strictly non-prescriptive (i.e., strictly not causal determinism). Intuitively, this is clear, since one should be able to choose freely, even to act predictably. Not to be able to choose to act predictably would imply a constraint on freedom.

The instance of Saintly determinism does not make our model compatibilistic, as it must be clear. In particular, other situations are allowed to exist by our model, that resist compatibilistic interpretation, as noted below. Suppose that an agent’s FW is not maximal. Then his choice will fluctuate randomly between choosing according to the dictates of his nature and the recommendation of his Understanding. This is illustrated in the following proposition, whose intuitively apparent truth is validated rigorously by our definition of FW.

**Proposition 2** *Presented with the choice between the good and the evil, the Conscientious Criminal vacillates.*

This criminal, being conscientious, has a clear Understanding of the virtue of ethical behavior, but, owing to lack of sufficiently high FW, cannot always overcome the compulsion of his criminal nature. His choice is random, being good sometimes and being evil at other times. Thus the randomness of his choice does not imply lack of free will. Here it

only implies low free will. The *pessimist* [2] implication [D] in SWFP is also therefore falsified. The recognition of FW as a new form of causation also clarifies why the first version of SWFP fails. For example, implication [VI] is incorrect for reasons stated above.

In contrast to Proposition 1, predictable behavior can arise also because of low degree of FW. Consider:

**Proposition 3** *Presented with the choice between the good and the evil, the Hardcore Criminal chooses evil.*

Although the Hard-core Criminal may possess an ethical Understanding, yet, because he almost entirely lacks FW, his choice is deterministically decided by his evil nature. Knowing that a person is a Hardcore Criminal, we have foreknowledge of the iniquity of all his future choices because he *lacks* any FW. If we theoretically empower him with some FW, we obtain the scenario of Proposition 2. In other words, adding a degree of FW removes determinacy. Clearly, this prediction of the model is not in accord with compatibilism. Therefore, our model is neither incompatibilist nor compatibilist.

## VI. SOME NEUROSCIENTIFIC CONSIDERATIONS AND PHILOSOPHICAL MUSINGS

The FUN model as it stands does not explain the notion of moral responsibility. When the given an AP situation is a moral one, responsibility arises arguably because a person has the power **F** to follow through the guidance provided by **U** by overcoming the dictates imposed by **N**. The model does not explain how the differences in the degree of freedom, eg., between that of a Saint and a Conscientious Criminal, arise. Unless an agent can somehow be construed as having played a role in the current level of freedom, the judgment of moral responsibility must be relative (to a given degree of freedom). Further assumptions are needed, which will be taken up in a subsequent work, to develop a more complete notion of moral responsibility in the model.

For the triad of elements of the FUN model described in section V, we suggest the following plausible brain correlates. For convenience, we refer to this neurologically adapted version of FUN as the FUN# model. The element **N**, and hence the mind's brain correlate, is probably the limbic system, which, comprising brain structures like the hippocampus, amygdala, etc., is known to support a variety of functions, including behavior, emotion and long term memory. The element **U**, and by extension the intellect's brain correlate is located presumably in the brain prefrontal cortex. This region of the brain is believed by neuroscientists and psychologists to be responsible for many higher cognitive functions, such as planning, differentiating between the good and bad, determining consequences of actions, planning actions, and so on.

The FUN# model posits that, while these two regions participate in decision making processes, they do not correspond to the *ultimate control module* (UCM), the brain correlate of FW. This could be a localized region, such as the pineal gland. The claim for this gland is mainly historical. The FUN# model accepts this as a working hypothesis on the strength of the observation that this gland is centrally located in the brain, as befits a UCM. Physiologically, this would mean that brain areas corresponding to voluntary muscle movement (the motor cortex) will be functionally linked to the frontal cortex, the limbic system and the region of the pineal gland, during the execution of planned actions.

In the FUN model, **F**, **U** and **N** are independent elements that interact when a choice is made. In actual fact, they influence each other at the *subconscious* level in complicated situation-dependent ways, so that the elements as they appear at the conscious level are 'dressed versions' rather than the 'bare versions' (to use terminology borrowed from optics or particle physics). The agent's personality is thus a nonlinear functional of the elements, where each element influences the others and also itself indirectly through its effect on the others.

An instance where **U** affects **N** is in consumers' brand loyalty, which manifests as preferences, detectable as neural correlates, that humans display towards products that are otherwise similar (cf. an interesting study reported in Ref. [28] with regard to drinks). An instance where **N** affects **U** is the familiar case of people showing reduced objectivity when dealing with those they have a strong emotional connection (whether negative or positive) with. An instance where **N** affects (diminishes) freedom is when a drug addict (resp., smoker) relentlessly seeks his next shot (resp., puff) even though he knows it is not good for him; and so on.

For this reason, from a behavioral perspective, it would be more convenient to talk not in terms of interaction between the elements, but instead in terms of interaction between three broad traits: that dominated by Nature **N** ("the animalistic"), that dominated by freedom ("the saintly"), and that wherein these two are in balance ("the human"). The philosophy of *Yoga* [24] terms these three basic traits as *tamas*, *rajas* and *sattva*. In this line of thought, a human character is constituted by these three basic traits and determined by their relative dominance. The *sattvic* type of humans have the highest degree of FW, while the *tamasic* have the lowest.

To complete the model, we offer some thoughts on the extraphysical agency to which the will is ascribed. Here we will appeal to the philosophies of *Vedanta* [24] and *Yoga*, which recognize and supply such an entity— the *Self* (*ātman* in Sanskrit), which is quite distinct from the Ego (*ahamkāra*) and conceived of as the essential individual. At a deeper

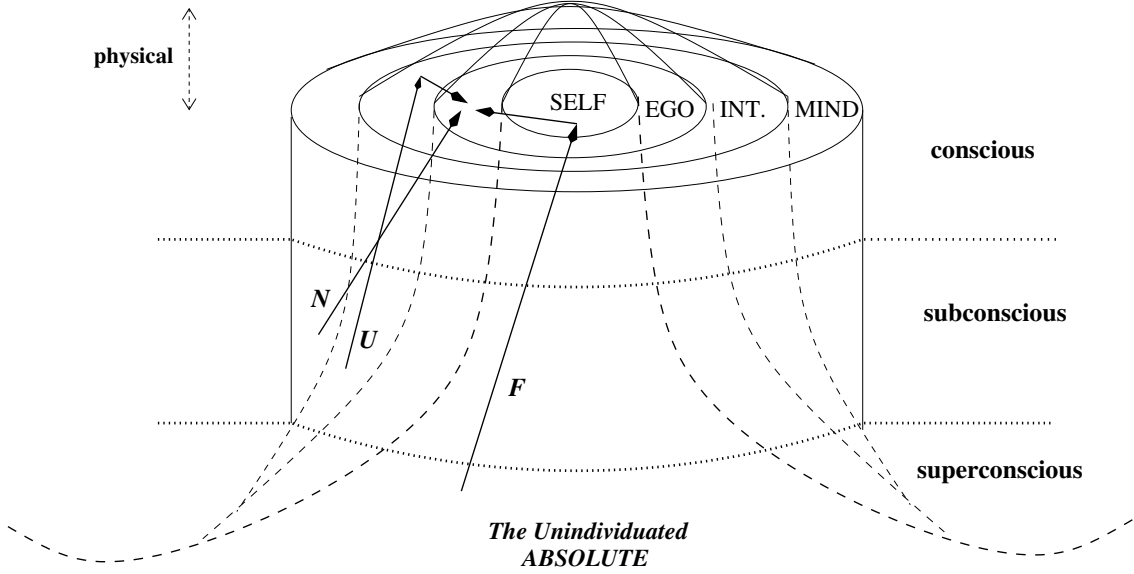


FIG. 1: Scheme of the structure of the individual psyche as used in our model of FW. The scheme is based primarily on the Eastern philosophy of *Vedanta* [24] and *Yoga* [24], with inputs from Freud’s iceberg model in psychoanalysis and E. Cayce [29]. The ego receives causal constraints from the mind, guidance from the intellect and freedom from the Self to direct the choice in line with the guidance. In a word: *Thought over Nature*. In this geometric interpretation, the ego consciousness is *horizontal* (spreading out concentrically from the center), while the mode of Cognizance of the Self is the *witness consciousness* (*sákshí bháva* in Sanskrit). This represents a bottom-up perspective from the base of the figure, which is *vertical* and illumines all overlying elements of the personality, including the ego. In this sense, the Self’s Witness consciousness is higher dimensional than the personality’s ego consciousness. (An even higher dimensional consciousness can be associated geometrically with an observer looking at the above image from outside the page. This perspective represents a subtler, Witness of Witness Consciousness. And so on. Vedantic philosophy posits that this hierarchy terminates in *brahman*, the Absolute or the fundamental substratum.)

Faculty	Seat	Function	Possible physical correlate	Character trait
Nature, <b>N</b>	Mind/Id	Mental constraints	limbic system	<i>tamas</i>
Understanding <b>U</b>	Intellect/Superego	Guidance	Brain frontal cortex	<i>rajas</i>
Freedom <b>F</b>	<i>Self</i> / –	Control	pineal gland?	<i>sattva</i>

TABLE I: The three structural elements of libertarian FW according to the FUN model, their corresponding place of origin (with a Freudian near-equivalent term), the function of the elements, possible physical correlates in the brain, and (in the terminology of Yogic philosophy) the corresponding character trait.

level of consciousness, the Self fans out to serve as the overarching *substratum* on which the agent’s faculties of Ego, Understanding and Nature rest. At the deepest level of consciousness, there is no structure and the individual Selves resolve into an unindividuated Absolute. Contradictory as these descriptions seem, they can be readily interpreted geometrically, cf. Figure 1. As in the Freudian *structural iceberg model*, the Constraints of **N** are communicated from the subconscious mind to the conscious Ego, where they are experienced as desires. The Understanding and Will are exercised *preconsciously*, in that although having deeper roots, they are supplied from within the conscious level.

The Self cognizes the faculties of Ego, mind and intellect through a mode of Consciousness called *witness consciousness* (*sákshí bháva*). This is subtler than and different from the agent’s Ego consciousness. Figure 1 depicts a simple geometric interpretation of these modes of awareness. The basic information about the the above three elements are summarized in Table I.

## VII. A QUANTITATIVE VERSION OF THE FUN MODEL

We develop a minimal mathematical model, denoted FUN+, to quantify the qualitative ideas developed above as part of the FUN model. Given an AP situation, and a choosing event **e**, let the choice be represented by a random variable *E* over the sample space  $\Omega \equiv \{e_1, e_2, \dots, e_n\}$ , representing the possible *n* choices that can be made. If **e**

involves a material particle, which presumably lacks any libertarian FW, the outcome  $e_j$  at event  $\mathbf{e}$  would occur with probability  $p_j$ , and there is no distortion of  $P$ .

The dictates imposed by the Constraints of Nature  $\mathbf{N}$  are represented, over  $\Omega$ , by the probability vector  $P = \{p_1, \dots, p_n\}$ , normalized so that  $\sum_{j=1}^n p_j = 1$ . The vector could be pure ( $\forall_j (p_j)^2 = p_j$ ) or mixed. The recommendation due to Understanding  $\mathbf{U}$  is represented, over  $\Omega$ , by the probability vector  $P^U = \{p_1^U, \dots, p_n^U\}$ , which is normalized and could be pure or mixed. For example, if  $\Omega = \{\text{good}, \text{evil}\}$ , then we would have  $P^U = \{1, 0\}$  according to the moral criterion, whereas by Nature,  $P = \{\frac{1}{2}, \frac{1}{2}\}$ . If  $\Omega = \{\text{coffee}, \text{tea}, \text{alcohol}\}$ , the Understanding could use a health criterion to return  $P^U = \{\frac{1}{2}, \frac{1}{2}, 0\}$ , whereas it may be that  $P = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\}$ .

FW  $\mathbf{F}$ , as freedom to drive the choice from the prescriptive  $P$  towards the recommended  $P^U$ , is represented by the scalar quantity  $\sigma$  (where  $0 \leq \sigma \leq 1$ ) such that greater freedom connotes larger  $\sigma$ . The probability vector  $P'$  representing the eventual choice of an alternative, is obtained by distortion of  $P$  towards  $P^U$  in the measure of strength of  $\sigma$ . A particularly simple form to capture this idea is a convex combination of  $P$  and  $P^U$ , parametrized by  $\sigma$ :

$$P' = \sigma P^U + (1 - \sigma)P. \quad (1)$$

Thus the random variable  $\mathcal{P}'$  representing the selected option is a weighted mean of random variables  $\mathcal{P}^U$  and  $\mathcal{P}$ , representing, respectively, Understanding and Nature. When  $\sigma = 1$  (maximal FW),  $P' = P^U$ , i.e., the choice is aligned with the Understanding. When  $\sigma = 0$  (vanishing FW)  $P' = P$ , i.e., the choice is entirely determined by Nature. For example, if  $P$  is the probability for obtaining an outcome 0 or 1 when measuring the Pauli observable  $\sigma_z$  on a quantum two-level system (qubit) in the state  $\alpha|0\rangle + \sqrt{1 - |\alpha|^2}|1\rangle$ , then  $P = (|\alpha|^2, 1 - |\alpha|^2)$ . Since the qubit arguably lacks FW ( $\sigma = 0$ ),  $P' = P$  according to Eq. (1). The nonlinear functional  $P'[P]$  must be viewed as a set of ontic probabilities, and thus a fundamental limitation on the predictability of an agent's choice, and not epistemic probabilities arising from ignorance about the details of an agent.

The unpredictability  $\xi$  of an agent may be quantified by the Shannon entropy of  $P' = \{p'_x\}$ :

$$\xi \equiv H(P'; \sigma) = - \sum_x p'_x \log p'_x = \langle \log(\sigma p_x^U + (1 - \sigma)p_x) \rangle, \quad (2)$$

where  $H(P'; \sigma)$  is the Shannon entropy of  $P'$  for a given value of  $\sigma$ . The following two results quantitatively present our earlier resolution of SFWP. In each case, we provide an example in the following two theorems, that contradicts each of implications **[C]** and **[D]** of SFWP.

**Claim 1** *The predictability of a Saint's behavior does not imply his lack of FW.*

**Proof.** We construct an explicit instance of predictable behavior with (high) FW. Let  $\Omega = \{\text{good}, \text{evil}\}$ . By definition, the Saint of Proposition 1 is ethical in his Understanding, so that  $P^U = (1, 0)$ , and his will is free, so that  $\sigma \approx 1$ . Even if he may bodily not be attuned to perfection, still by dint of his high FW, he is always able to choose in accord with his (ethical) Understanding, i.e.,  $P' = P^U$  by Eq. (1). From Eq. (2), we have  $H(P'; \sigma) \approx 0$  bit, implying almost complete predictability, irrespective of  $P$ . ■

This implies, as noted earlier, that our model is not incompatibilist. More generally, we note that  $H(P'; \sigma)$  is not a monotonous function of  $\sigma$ . The regime where

$$\frac{dH(P'; \sigma)}{d\sigma} < 0, \quad (3)$$

and thus increase in FW leads to certainty, may be regarded as a zone of disagreement with the incompatibilist position. For example, given a situation involving two choices, with  $\Omega = \{0, 1\}$  and  $P^U = \{1, 0\}$  and  $P = \{0, 1\}$ ,  $dH/d\sigma < 0$  for  $0.5 < \sigma \leq 1$ .

**Claim 2** *The randomness of the Conscientious Criminal's choice does not imply his lack of FW.*

**Proof.** We construct an explicit instance of random behavior with non-vanishing FW. In the case of the Conscientious Criminal of Proposition 2, who has non-vanishing but not maximal FW, let  $\sigma = \frac{1}{2}$ , and let  $P \approx (0, 1)$ , corresponding to the constraint imposed by his evil Nature. From Eq. (1), we have  $P' = (0.5, 0.5)$ , and hence  $\xi \approx 1$ , implying near maximal randomness in choice. However, he does not lack FW, as  $\sigma > 0$ . ■

In the case of the Hard-core Criminal of Proposition 3, we have the same  $\Omega$ ,  $P$  and  $P^U$  as in the Example used in Theorem 2, but  $\sigma \approx 0$ , corresponding to this criminal's low FW. From Eq. (1), we have  $P' \approx P$ , and thus from



Eq. (2),  $\xi \approx 0$ , implying very little unpredictability in choice, similar in this respect to the Saint of Proposition 1. However, unlike with the saint, whose predictability comes from high FW, here the predictability is due the criminal's sure surrender to his natural instinctive constraints. Augmenting his FW, we find that

$$\frac{H(P'; \sigma)}{d\sigma} > 0, \quad (4)$$

that is, increase in FW leads to increase in uncertainty (when  $0 < \sigma < 0.5$ ). Thus our model is also not compatibilist. Together, Eqs. (3) and (4) imply that our model is neither compatibilist nor incompatibilist.

The FUN model implies that  $\sigma$  must be at least partially extra-physical and cannot be described by any purely physical theory, even a theory-of-everything (ToE), that encompasses only physical phenomena. In particular, artificial intelligence (AI), which is fundamentally built on (quantum) physical rules, cannot capture true cognitive behavior.

These considerations entail that a human agent, and by extension any sentient agent, could not be considered merely as a sufficiently complex robot, but a qualitatively distinct class of entities. Here Penrose's interesting thesis is worth noting, according to which conscious processes are fundamentally non-algorithmic [30].

### VIII. QUANTUM INDETERMINISM AND FREE WILL

WFWP shows that quantum indeterminism is not better for FW than classical determinism. However, the fact remains that the world is fundamentally a quantum mechanical place. Further, there is in a sense a lesser departure from the physical dynamics  $\mathcal{D}$  through free-willed intervention if  $\mathcal{D}$  were indeterministic rather than deterministic, in that there is only a *statistical* violation of causality in the former case, rather than the *logical* violation of causality, as it is with the latter.

Accordingly, if  $X$  is the random variable corresponding to a FW-influenced quantum measurement  $\mathcal{M}$  on system  $S$  (presumably a suitably small brain element) in state  $|\psi\rangle$ , then  $X$  would deviate from the Born probability rule. Free-willed intervention will therefore manifest as *statistical deviations from the Born rule*. By contrast, the freedom or 'free will' of quantum particles, which is plain quantum randomness, will conform to the Born rule. In the model described below (called FUN++), which is a particular realization of the FUN+ model, we propose that FW intervenes by controlling the collapse of the wave function.

To begin with, we represent the AP situation as a *uniform* quantum superposition, and the willful choice by a directed collapse of the wavefunction. This quantum model thus presumes objective collapse of the wave-function, and is compatible with interpretations of quantum mechanics that admit it. The exercise of FW is broadly divided into three stages as follows:

**Attention.** Faced with an AP situation of alternatives  $j$ , the brain creates a quantum superposition that reflects the dictates of Nature **N**:

$$|\Psi\rangle = \sum_j \sqrt{p_j} |j\rangle \quad (5)$$

in a suitable subneuronal system  $S$  in an appropriate basis  $\mathcal{B} = \{|j\rangle\}$ , where the states  $|j\rangle$  correspond to the base choice space  $\Omega$ . The main requirement is that it should be possible to shield  $S$  indefinitely from decoherence or other noise, while the process of making a choice is under way. According to Ref. [27], brain microtubules may be the seat of such superpositions.

**Survey.** The ego surveys the recommendations of the mind and intellect. This event is mathematically represented by the preparation of a nonlinear positive operator-valued measurement (POVM)

$$M_j \equiv \sqrt{p_j^{-1} (\sigma p_j^U + (1 - \sigma) p_j)} |j\rangle \langle j|, \quad (6)$$

which satisfy the completeness condition  $\sum_j \langle \Psi | M_j^\dagger M_j | \Psi \rangle = 1$ . In the absence of FW,  $\sigma = 0$ , and the POVM reduces to ordinary projective measurement.

**Collapse.** Application of the POVM to the state  $|\Psi\rangle$  causes the transition:

$$|\Psi\rangle \longrightarrow \frac{M_j |\Psi\rangle}{\sqrt{\langle \Psi | M_j^\dagger M_j | \Psi \rangle}}, \quad (7)$$

with probability  $\langle \Psi | M_j^\dagger M_j | \Psi \rangle$ .

The last stage in the model is tantamount to controlling and directing the wavefunction collapse to produce an outcome probability distribution that is closer (as quantified by trace distance, relative entropy or any other suitable distance measure) to  $P^U = \{p_j^U\}$  than  $P = \{p_j\}$ , if FW is available. If FW is absent, then Nature alone determines the outcome probability distribution, which, moreover, conforms to the Born rule.

When  $\sigma \neq 0$ , the resulting probabilities of outcomes will violate the Born rule. In general, such violations can give rise to various non-standard effects, among them violation of energy conservation, possibility of signaling at superluminal speed [31] and the possibility of counter-intuitive models of computation that allow efficient solution of hard problems [31]. However, these non-standard effects will be confined to a small sub-cellular region of the brain, where it will not be easily discernible from measurement errors, decoherence effects, neural noise and statistical fluctuations. Moreover, it can be easily masked behind the remaining features of the brain's physiology, which can be described in terms of deterministic, classical mechanisms (e.g., metabolic changes leading to arousal potentials in motor neurons, initiating familiar physical movements of the body).

Finally, it is worth stressing that the extra-physical agency posited to determine the outcome of wavefunction collapse should not be confused with hidden variables in the sense of Bohm [32]. The latter represents a device to turn QM into a deterministic theory, and thus restore classical causality, whereas in our approach requires, FW is a new form of causation.

## IX. NEUROSCIENTIFIC IMPLICATIONS, TESTS AND APPLICATIONS

In the conventional view, the brain is a complex input-output device, with apparently voluntary actions arising from a mix of deterministic and indeterministic mechanisms. So it is an interesting question what would constitute a falsifiable proof of FW. For example, in research reported in Ref. [33], tethered fruit flies (*Drosophila melanogaster*) in a visually impoverished environment had their flight maneuvers recorded. Lacking any external input, their random behavior should have resembled a Poisson process. However, the analysis showed that the temporal structure of fly behavior follows a Lévy-like probabilistic behavior pattern. At first blush, it seems possible that this may be due to spontaneous behavior arising from (a primitive version of) libertarian FW of the fruit flies. However, it turns out that the observed pattern can be simulated via intrinsic noise amplified by suitable nonlinearity [33].

Experiments that purport to *disprove* the existence of FW can also find alternate explanations. Based on a study of volunteers wearing scalp electrodes, Libet and collaborators [34] showed in 1983 that a 'readiness potential' (RP) was detected a few tenths of second before the subjects, in their own reckoning, made the decision to perform an action (to flex a finger or wrist). Their result was interpreted as indicating that the motor cortex was preparing for the action, that unconscious neural processes predetermine actions and hence that FW was illusory. In a recent comment on the experiment, Miller and Trevena [35] asked subjects to wait for an audio tone before making a decision to tap a key or not. If the activity detected by Libet et al. really was the making of the decision prior to any conscious awareness of doing so, then that activity ought to occur only if the subjects decided to act. But no such correlation was found. Miller and Trevena conclude that the RP may only indicate that the brain is paying attention and does not indicate that a decision has been made.

Under the circumstances, one suggestion would be to trace backwards along a deterministic observable causal chain of neurons to the specific area in the motor cortex responsible, perhaps a single neuron, that may be considered as initiating a free-willed action at the physiological level, and hence a candidate site for carrying signatures of the putative new physics, such as the claimed deviation from the Born rule.

Identifying such single neurons will be difficult, given the high density of neuronal packing, and the weakness of excitatory synapses, which would make the role of individual neurons in the brain cortex hard to identify. Further, the extreme complicatedness of brain dynamics and attendant decoherence, measurement errors, noise and statistical fluctuations, will play an adverse role. A study where the twitching of a mouse's whisker has been traced to single pyramidal neurons in the cortex is reported in Ref. [36]. These neurons could be examined for unusual behavior that might be considered as appropriate signatures, after suitable allowance has been made for noise and innervations from neighboring neurons: for example, there may be fluctuations in the whisker-twitch causing potentials that could somehow not be ascribed to any neighboring influences. The neuron may then be examined carefully for a sub-cellular site to which the initiation of the twitch causing potentials may be attributed.

Another promising area to look for evidence suggestive of FW is psychiatry and neurology, which are startlingly revelatory on how, from a physiological perspective, the human mind works [38]. The FUN# model suggests that the decision making ability in a human is affected, possibly leading to anti-social behavior, in broadly three *different* ways: when the limbic system, or the frontal cortex, or the pineal gland area (or any other candidate UCM) is impaired. If in some case such behavior does not coincide with impairment to the limbic system or the frontal cortex, then there is a reasonable case to attribute the behavior to diminished FW. Studying such cases, if any, would help locate the UCM and verify whether the pineal gland hypothesis holds good.

There are potential therapeutic and medical applications based on the above observations. The FUN# model can help clarify what abnormalcy of behavior means and can help classify abnormal depending of which faculty is affected. If a particular behavioral disorder can be attributed to one of the three above causes, then the therapy prescribed to the afflicted patient could also be varied with better efficacy.

## X. CONCLUSIONS AND DISCUSSIONS

A conservative, Darwinian view regarding the origin of FW would be that it enables an organism to deviate from the local gradient in the brain's energy landscape, and to better than locally optimize in the struggle for self-preservation, which would require improvisation under novel situations; once this was done, FW brought along with it potential by-products, like altruism, self-harm, etc., behaviors which are not necessarily conducive to self-preservation, and hence not 'intended' by the evolutionary process that gave rise to FW. Another possibility is that FW is an emergent phenomenon that is a product of self-organization in complex quantum physical systems. These views are not necessarily incompatible with the FUN model. If anything, one can be adventurous and suggest this extra-physical agency guides or conduces to Darwinian evolution to make the human physiology optimal for the physical experience! For example, it is known that the higher the oxygen concentration (up to a point), the better it is for the metabolism and life of organisms. It is also known that the lower the altitude, the higher the oxygen concentration. If there is FW, an organism in a mountainous area may feel the urge to explore farther by climbing higher, thereby improving its chances of reaching a valley that is deeper, due to which the oxygen concentration is higher, than in the valley reached by following the local gradient.

Finally we offer our opinion on FW being, together with awareness, aspects of consciousness. To be aware of a theory is to be able to talk about it and to understand its implications and limitations. To this end, one requires a meta-theory, which is equipped with a language in which to formalize propositions about the theory. An axiomatic system that is rich enough to encompass the meta-theory will, in general, be more powerful than the one that axiomatizes the theory. An instance of this is provided by Gödel's theorem [21], which provides a metamathematical proof of mathematically undecidable propositions.

If FW did not exist, and the behavior of observers were entirely determined by the rules of the base theory, then the observers' algorithmic complexity [37] would not be greater than that of the theory, making them incapable of encompassing the meta-theory. Equipped with FW, human experimenters can be *meta-entities* in the theory. It allows them to freely pose questions, perform tests and draw inferences about the theory as external agents whose choices are not entirely determined by the theory. From this perspective, FW is an aspect of the consciousness of sentient observers that is necessary to make them aware of a physical theory such as QM and to experience the (quantum) world as humans do.

- 
- [1] T. O'Connor. Free will. <http://plato.stanford.edu/entries/freewill/>
  - [2] K. Timpe. Free will. <http://www.iep.utm.edu/freewill/>
  - [3] L. Zagzebski. Foreknowledge and Free Will. <http://plato.stanford.edu/entries/free-will-foreknowledge/>
  - [4] Wikipedia entry of free will. [en.wikipedia.org/wiki/Free\\_will](http://en.wikipedia.org/wiki/Free_will)
  - [5] S. Esposito. A personal point of view about scientific discussions on free will. Eprint arXiv:1202.3395.
  - [6] J. Conway and S. Kochen. The Free Will Theorem. Found. Phys. 36 1441 (2006). arXiv:quant-ph/0604079.
  - [7] J. Conway and S. Kochen. The Strong Free Will Theorem. Amer. Math. Soc. 56, 2 (2008); arXiv:0807.3286.
  - [8] H. Stapp. Quantum theory and the role of mind in nature. quant-ph/0805.0116.
  - [9] G. 't Hooft. The Free-Will Postulate in Quantum Mechanics. quant-ph/0701097.
  - [10] H. Nikolić. Closed timelike curves, superluminal signals, and "free will" in universal quantum mechanics. arXiv:1006.0338.
  - [11] A. Suarez. Quantum randomness can be controlled by free will- a consequence of the before-before experiment. arXiv:0804.0871.
  - [12] A. Suarez. The General Free Will Theorem. arXiv:1006.2485.
  - [13] A. Suarez. The Free Will Theorem and the Flash Ontology implicitly assume the Before-Before Experiment and thereby imply free will. Comment on a note by Nicolas Gisin in arXiv:1002.1392v1. arXiv:1006.2485.
  - [14] V. Vedral, Is the universe deterministic? New Scientist, 18 November 2006, p. 55.
  - [15] N. Gisin. The Free Will Theorem, Stochastic Quantum Dynamics and True Becoming in Relativistic Quantum Physics. arXiv:1002.1392.
  - [16] N. Gisin. Are There Quantum Effects Coming from Outside Space-time? Nonlocality, free will and no many-worlds. quant-ph/1011.3440.
  - [17] M. J. W. Hall. Local deterministic model of singlet state correlations based on relaxing measurement independence. Phys. Rev. Lett. **105**, 250404, 2010.

- [18] S. Hossenfelder. The free will function. arXiv:1202.0720
- [19] G. Svetlichny. How Free Will Could Will. <http://arxiv.org/abs/1202.2007>.
- [20] M. Mackay. *The Clockwork Image: A Christian Perspective on Science* Inter-Varsity Press, London (1997).
- [21] K. Gödel. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I. Monatshefte für Mathematik und Physik 38 (1931). (Various translations/commentaries are available on the web.)
- [22] A. Turing. On computable numbers, with an application to the Entscheidungsproblem. Proc. London Math. Soc 42, 230 (1936).
- [23] J. D. Barrow. *Impossibility: The Limits of Science and the Science of Limits*. Oxford University Press, USA (October 28, 1999).
- [24] R. Srikanth. Srimadbhagavad Gita in the New Age (2012): <http://poornaprajna.org/srik/gitopanishad>
- [25] D. Hodgson. Constraint, Empowerment, and Guidance: a Conjectural Classification of Laws of Nature. Philosophy 76, 341-70 (2001). <http://users.tpg.com.au/raeda/website/constraint.htm>
- [26] R. Kane. [http://en.wikipedia.org/wiki/Robert\\_Kane\\_\(philosopher\)](http://en.wikipedia.org/wiki/Robert_Kane_(philosopher))
- [27] S. Hameroff and R. Penrose. Orchestrated Objective Reduction of Quantum Coherence in Brain Microtubules: The "Orch OR" Model for Consciousness. <http://www.quantumconsciousness.org/penrose-hameroff/orchOR.html>
- [28] S. M. McClure, J. Li, D. Tomlin, K. S. Cypert, L. M. Montague and P. Read Montague. Neural Correlates of Behavioral Preference for Culturally Familiar Drinks. Neuron 44, 379 (2004).
- [29] H. B. Puryear. The Edgar Cayce Primer: Discovering The Path to Self-Transformation (Bantam Books, 1982).
- [30] R. Penrose. Shadows of the Mind (Oxford University Press, 1994).
- [31] R. Srikanth. Entanglement, intractability and no-signaling. Physica Scripta 81 (2010) 065002.
- [32] D. Bohm and B. Hiley. The Undivided Universe: An ontological interpretation of quantum theory. (London: Routledge 1993).
- [33] A. Maye, C.- H. Hsieh, G. Sugihara and B. Brembs. Order in Spontaneous Behavior. PLoS ONE issue 5, e443 (May 2007). Available freely online from [www.plusone.org](http://www.plusone.org).
- [34] B. Libet, C. A. Gleason, E. W. Wright and D. K. Pearl. Time of conscious intention to act in relation to onset of cerebral activity (readiness-potential): the unconscious initiation of a freely voluntary act. Brain (1983) 106 (3): 623-642. doi: 10.1093/brain/106.3.623.
- [35] J. Trevena and J. Miller. Brain preparation before a voluntary action: Evidence against unconscious movement initiation. Consciousness and Cognition Volume 19, Issue 1, March 2010, Pages 447-456. doi:10.1016/j.concog.2009.08.006.
- [36] M. Brecht, M. Schneider, B. Sakmann, T. W. Margrie. Whisker movements evoked by stimulation of single pyramidal cells in rat motor cortex. Nature 427, 704 (2004).
- [37] Chaitin. Algorithmic Information Theory (Cambridge University Press, 1987).
- [38] V. Ramachandran and S. Blakeslee. Phantoms in the Brain (Harper Perennial 1999).